Shape Phase Evolution of the Axially Symmetric States Between the U(5) and SU(3) Symmetries

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Abstract

The shape phase structure and its transition of the nucleus in the transitional region between the U(5) and SU(3) symmetries is restudied in the framework of coherent-state theory with angular momentum projection in IBM-1. The certain angular momentum (or rotation-driven) effect on the nuclear shape is discussed. A coexistence of prolate and oblate shapes is found for the ground states of the transitional nuclei. A phase diagram in terms of the deformation parameter and angular momentum is given.

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The nuclear shape phase transition has been one of the most interesting and significant subject in the research of nuclear structure (see for example Refs.[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]). Then the ground state shape phase transition between the U(5) and SU(3) symmetries of the interacting boson model-1 (IBM-1) has been well studied [2, 3, 4, 6, 14]. The study using coherent state theory shows that there is a first order phase transition from spherical shape to axially deformed shape at a certain critical value of the control parameter [2] and there may involve coexistence of spherical and axially deformed shapes [3]. However, the result of using coherent state theory with angular momentum projection shows that there is no shape coexistence and the transition is continuous as the control parameter changes[4]. Even so, the minimum corresponding to the negative deformation parameter β has not yet been paid attention. On the other hand, an analytical solution, namely the X(5) symmetry, has recently been found for the states around the critical point of the phase transition from the U(5) symmetry to SU(3) symmetry using the collective model [6], where the potential is in an infinite square well about the shape variable β (β-soft). In this paper, we will restudy the ground state shape phase transition and the shape coexistence using the coherent state formalism with angular momentum projection. On the other hand, it has been found in experiment that rotation (or angular momentum) may induce a shape phase transition from the spheroid to the axial ellipsoid[10]. Since the angular momentum projection can extend the coherent state formalism to study the shape of excited states with certain angular momentum and has been successful in describing the symmetry of the critical point in the transition from U(5) symmetry to O(6) symmetry[12], we will also study the rotation-driven shape evolution along the ground state band of the transitional nuclei in the region of U(5)-SU(3) symmetries in this paper.

To study the shape phase transition, one takes usually the parametrized Hamiltonian in the framework of the IBM-1

$$\hat{H} = c\left(\eta \hat{n}_d - \frac{1-\eta}{N}\hat{Q}(\chi)\cdot\hat{Q}(\chi)\right), \tag{1}$$

where $\hat{n}_d$ is the d-boson number operator and $\hat{Q}(\chi) = [s^\dagger\tilde{d} + d^\dagger s]^{(2)} + \chi[d^\dagger\tilde{d}]^{(2)}$ is the

quadrupole operator. The c just scales the interaction strength. The N in the denominator is the total boson number[4, 18]. Eq.(1) can also be written as

$$\begin{aligned}
\hat{H} &= \left[\eta - \frac{2(1-\eta)\chi}{7N}\Big(\chi + \frac{\sqrt{7}}{2}\Big)\right] cC_{1U(5)} - \frac{2(1-\eta)\chi}{7N}\Big(\chi + \frac{\sqrt{7}}{2}\Big) cC_{2U(5)} \\
&\quad - \frac{2(1-\eta)}{\sqrt{7}N}\Big(\chi + \frac{\sqrt{7}}{2}\Big) cC_{2O(6)} + \frac{2(1-\eta)(\chi+\sqrt{7})}{7N}\Big(\chi + \frac{\sqrt{7}}{2}\Big) cC_{2O(5)} \\
&\quad + \frac{(1-\eta)\chi}{\sqrt{7}N} cC_{2SU(3)} - \frac{(1-\eta)\chi(\chi+2\sqrt{7})}{14N} cC_{2O(3)}\,, \qquad (2)
\end{aligned}$$

where C_{kG} is the k-rank Casimir operator of group G. It is evident that, at certain points of the parameter space, the above Hamiltonian reaches the dynamical symmetry limits of the IBM-1. If $\eta = 1$ and χ arbitrary, it gives the U(5) symmetry without two-body interactions being included; if $\eta = 0$ and $\chi = -\sqrt{7}/2$, it is in the SU(3) symmetry; if $\eta = 0$ and $\chi = 0$, it possesses the O(6) symmetry. Meanwhile, one can realize the transition from the U(5) symmetry to the SU(3) symmetry by fixing $\chi = -\frac{\sqrt{7}}{2}$ and varying η from 1 to 0.

To study the geometric content of the system with Hamiltonian in Eq.(1), we take the intrinsic coherent state $|g\rangle$ [2]

$$|g\rangle = (b_g^\dagger)^N |0\rangle, \qquad (3)$$

with

$$b_g^\dagger = s^\dagger + \beta\cos\gamma d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma(d_{-2}^\dagger + d_2^\dagger)\,,$$

where $s^\dagger$, $d_\mu^\dagger$ are the boson creation operators, $|0\rangle$ is the boson vacuum, and β, γ are the Hill-Wheeler intrinsic variables. It has been shown that the β and γ here are proportional to the deformation parameters $\hat{\beta}_2$ and $\hat{\gamma}$ in the collective model (e.g., for rare earth nuclei, $\hat{\beta}_2 \approx 0.15\beta$ and $\hat{\gamma} = \gamma$)[2]. Therefore one usually refers the β and γ here to deformation parameters for simplicity. Since the U(5) and SU(3) symmetries in classical limit correspond to spherical and axially deformed shapes[2], respectively, we can take only the axially symmetric case, where $\gamma = 0$ or $\gamma = \pi$, into account. To the convenience of description, we take $\gamma = 0$ and let β be able to be positive or negative, corresponding to the prolate or oblate shape, respectively[13]. Then we can obtain the equilibrium shape of

the ground state by evaluating the potential energy surface $E(N,\beta)$ at the above intrinsic coherent state and minimizing the $E(N,\beta)$ with respect to β. After some derivation, $E(N,\beta)$ can be given as

$$\begin{aligned} E(N,\beta) \;=\; & \frac{cN}{(1+\beta^2)^2}\left\{\frac{5(\eta-1)}{N}+\left[(5\eta-4)+\frac{(\eta-1)(2+\chi^2)}{N}\right]\beta^2\right. \\ & \left. +4\sqrt{2}(N-1)\frac{(1-\eta)\chi}{\sqrt{7}N}\beta^3+\frac{7[(N+1)\eta-1]+(5+2N)(\eta-1)\chi^2}{7N}\beta^4\right\}. \end{aligned} \tag{4}$$

In order to obtain the potential energy surface of the nuclear states with certain angular momentum L, we take the angular momentum projection on the coherent state[19, 20, 21]. Generally the angular momentum projection operator is written as[20, 21, 22]

$$P^L_{MK}=\frac{2L+1}{8\pi^2}\int D^{*L}_{MK}(\Omega)R(\Omega)d\Omega\,, \tag{5}$$

where $R(\Omega)$ is the rotational operator, $D^L_{MK}(\Omega)$ is the rotational matrix, and Ω is the Euler rotational angle (α',β',γ'). Taking advantage of the phenomenological IBM, one knows that the states in the ground state band are the ones with Z-component $K=0$ in the intrinsic frame. Then to study the ground state band, we limit the P^L_{MK} to P^L_{00}. The potential energy functional of the states in the ground state band (or the yrast band) can thus be expressed as

$$E(N,L,\beta)=\frac{\langle g|\,HP^L_{00}\,|g\rangle}{\langle g|\,P^L_{00}\,|g\rangle}\,. \tag{6}$$

With the explicit form of $R(\Omega)$ being substituted into, we obtain the potential energy functional (in fact, since what we consider at present is only the case at zero temperature, such a potential energy functional is just the free energy) as

$$E(N,L,\beta)=\frac{\int d\beta'\sin\beta' d^L_{00}(\beta')\,\langle g|\,\hat{H}e^{-i\beta' J_y}\,|g\rangle}{\int d\beta'\sin\beta' d^L_{00}(\beta')\,\langle g|\,e^{-i\beta' J_y}\,|g\rangle}\,, \tag{7}$$

where $d^L_{00}(\beta')$ is the reduced rotational matrix. Noting that $d^L_{00}(\beta')=P_L(\cos\beta')$, where P_L is the Legendre polynomial, we can accomplish the integrations and obtain the explicit expression of the free energy numerically. Meanwhile, It is easy to find that $E(N,L,\beta)=0$ if $L>2N$. Such a result is consistent with what the phenomenological IBM gives. Therefore we take the angular momentum $L\in[0,2N]$ in the following discussion. To

discuss only the axially symmetric states in the transitional region between the U(5) and SU(3) symmetries, we vary the parameter η and fix the angular deformation parameter $\gamma \equiv 0$ and the parameter $\chi = -\frac{\sqrt{7}}{2}$.

Ref.[4] has taken the nucleus with total boson number $N = 10$ as an example to study the ground state shape phase transition using Eq.(4). It shows that the potential energy surface involves a double minimum structure, corresponding to spherical and axially deformed shapes, respectively, in a very narrow region of control parameter, namely $0.771 \leq \eta \leq 0.795$[3, 4]. Then a first order shape phase transition happens when the global minimum changes discontinuously from $\beta = 0$ (spherical shape) to $\beta > 0$ (deformed shape) at $\eta = 0.793$. Because the barrier between the two minima is shallow, the coexistence of spherical and deformed shapes can appear. Our restudy at present reproduces this result well, which is illustrated in the $\beta \geq 0$ part of Fig. 1. As the certain angular momentum effect is involved by taking the angular momentum projection, the study in Ref.[4] using Eq.(7) shows that no double minimum structure is present and the global minimum shifts continuously from $\beta = 0$ for $\eta = 1$ to $\beta > 0$ for $\eta < 1$, whereas a local maximum develops at $\beta = 0$. Our restudy reproduces such a result well, too, as shown in the $\beta \geq 0$ part of Fig.2.

However, the above study is limited to $\beta \geq 0$. In fact, the value of the deformation parameter β can be negative, which corresponds to a oblate shape. So it is necessary to include the case of $\beta < 0$ to give a complete understanding of shape phase structure and its transition of the nuclei in the between of U(5) and SU(3) symmetries. Then we show the complete feature of the potential energy surface expressed in Eqs.(4) and (7) in Figs.1 and 2, respectively. From Fig.1, one can recognize that the results does not change except that, after the local minimum at $\beta = 0$ becomes a local maximum, a local minimum at $\beta < 0$ develops and changes continuously as η decreases. Since the minimum at $\beta > 0$ is much lower than that at $\beta < 0$, the coexistence of prolate and oblate shapes is not likely to appear. The result illustrated in Fig.2, however, shows that, if the control parameter $\eta > 0.850$, the difference of the two minima corresponding to $\beta > 0$, $\beta < 0$, respectively, is quite small and the barrier between the minima is shallow. Then the prolate and oblate

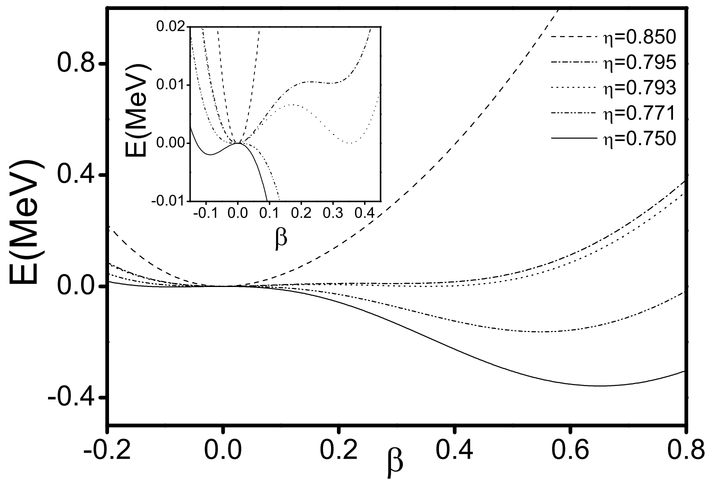

Figure 1: Potential energy surface $E(N, \beta)$ for the nuclei with total boson number $N = 10$. Five curves are plotted for different values of the control parameter η as the same as those in Ref.[4].

deformation shapes may coexist. Actually, as the control parameter $\eta \to 1$, the values of $E(N, L = 0, \beta)$ at the two minima become more and more close and coincide at $\eta = 1$, where $\beta = 0$. It seems that no first order shape phase transition occurs, because the global minimum at $\beta > 0$ changes continuously with the control parameter η.

With various angular momentum $L = 0, 2, 4, 6, \cdots$ being taken in Eq.(7), we can study the rotation (angular momentum) driven effect on the nuclear potential energy surface. The results for the U(5) symmetry limit with the two-body interactions being neglected (with $\eta = 1$), the SU(3) symmetry limit (with $\eta = 0$) and the interplay between the U(5) and SU(3) symmetries (taking the case with $\eta = 0.850$ as an example) are illustrated in Figs.3, 4, 5, respectively. Fig.3 shows obviously that all the $E(N, L, \beta)$s with different angular momentum L have the same minimum at $\beta = 0$. It means that rotation does not change the structure of the potential energy surface $E(N, L, \beta)$ of the states in U(5) symmetry without two-body interactions, so that the states maintain vibrational ones and the nucleus appears as a spheroid. However, the β-soft[13] disappears as $E(N, L, \beta)$

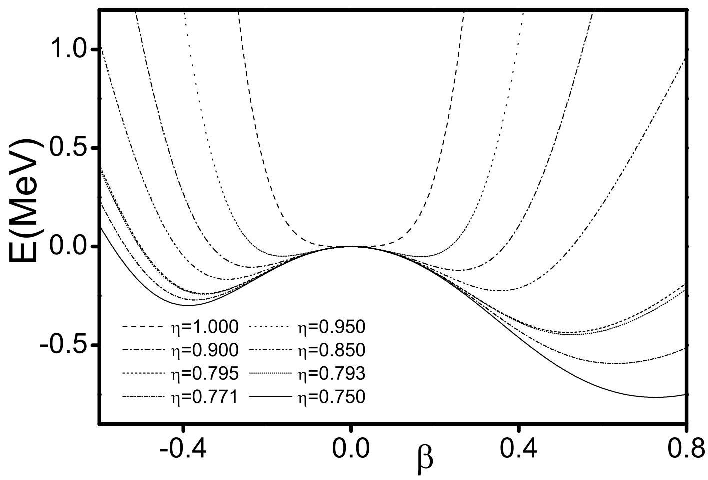

Figure 2: Effect of varying the control parameter η on the potential energy surface $E(N, L, \beta)$ for the nuclei with total boson number $N = 10$ and angular momentum $L = 0$.

becomes steeper around $\beta = 0$ if the angular momentum takes a nonzero value. From Fig.4, one can recognize that, the potential energy surface $E(N, L, \beta)$ of the states with low angular momentum and SU(3) symmetry involves a global minimum at $\beta = \sqrt{2}$ and a local minimum at $\beta < 0$. Corresponding to the increase of angular momentum, the local minimum at $\beta < 0$ gets shallowed, and then disappears. It indicates that the stable shape of all the states in the SU(3) symmetry always appears as an prolate ellipsoid. Furthermore, such a prolate shape gets more stable and the metastable oblate shape disappears with the angular momentum increasing. For the transitional nuclei, for example the one with control parameter $\eta = 0.850$, from Fig.5, one can infer that the prolate and oblate shapes may coexist at the state with angular momentum $L = 0$, since the values of the two minima at $\beta > 0$ and $\beta < 0$ are quite close to each other. As the angular momentum increases, the minimum at $\beta < 0$ becomes shallower and eventually disappears, meanwhile the minimum at $\beta > 0$ gets deeper and the corresponding β increases. It indicates that rotation breaks the shape coexistence and makes the deformation of the stable shape bigger and bigger. As a critical angular momentum is reached, the nucleus appears only

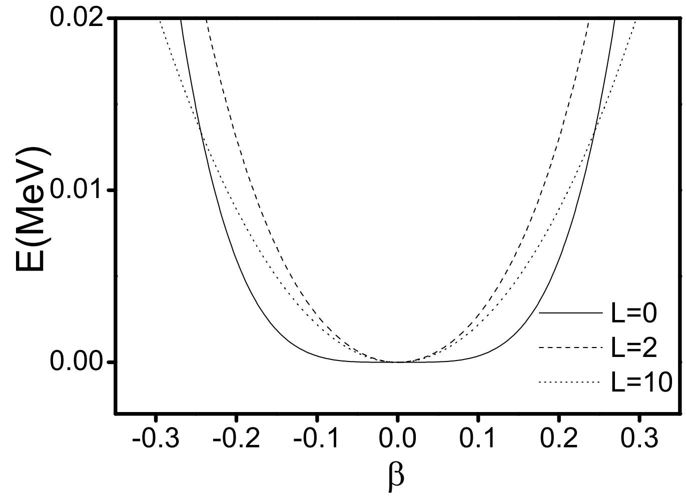

Figure 3: The potential energy surface $E(N, L, \beta)$ for the states in the U(5) symmetry with boson number $N = 10$ and angular momentums $L = 0, 2, 10$.

in a prolate ellipsoid shape. If the shape coexistence can be taken as a special shape phase, we can say that a shape phase transition from coexistence to prolate shape occurs. And the phase transition may be continuous.

To show the characteristic of the shape evolution of the transitional nuclei in the region from U(5) symmetry to SU(3) symmetry against the angular momentum L more clearly, we illustrate the phase diagram in terms of the deformation parameter and angular momentum at several values of the control parameter η in Fig.6. From Fig.6 and Fig.4, one may conclude that the nucleus with SU(3) symmetry is like a rigid rotor, the shape of which is so hard as not to be affected by the rotation and without any vibrational freedom. Then the energy spectrum of SU(3) symmetric nucleus is in good rotational. By the way, it should be mentioned that the deformation parameter β ($=\sqrt{2}$) of the states in SU(3) symmetry and with certain angular momentum is just the same as that of the classical limit (total boson number $N \to \infty$) in the case of without certain angular momentum. Whereas, the nucleus with U(5) symmetry is like a spheroid, and such a shape is not influenced by the rotation, either. Therefore the spectrum of the nucleus in

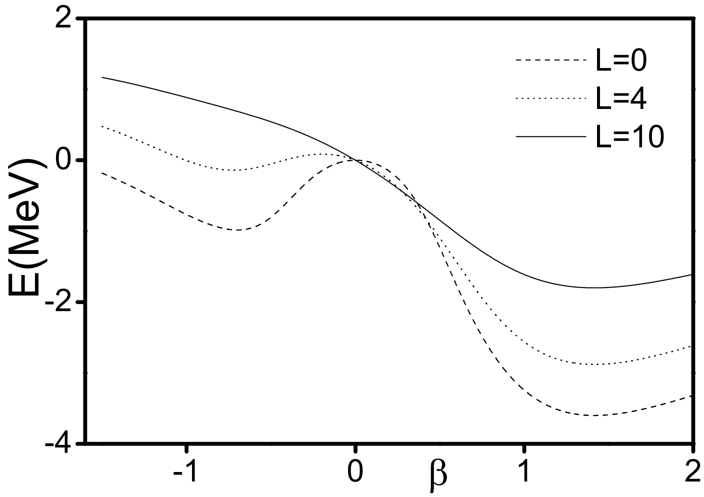

Figure 4: The potential energy surface $E(N, L, \beta)$ of the states in the SU(3) symmetry with boson number $N = 10$ and angular momentums $L = 0, 4, 10$.

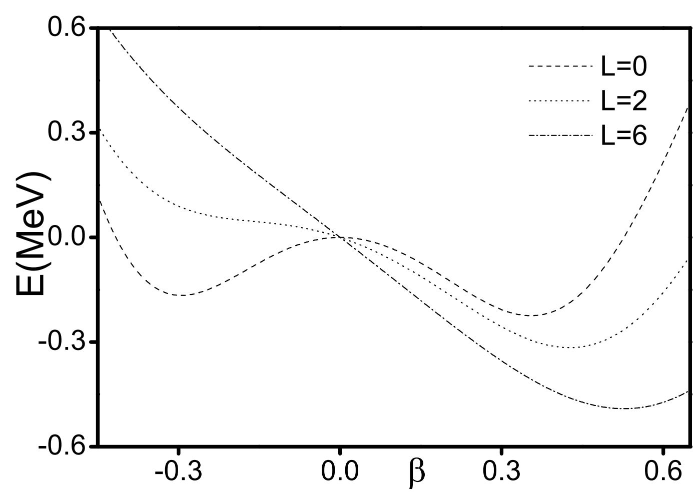

Figure 5: The potential energy surface $E(N, L, \beta)$ for the boson number $N = 10$ and angular momentum $L = 0, 2, 6$ states of the transitional nucleus with control parameter $\eta = 0.85$.

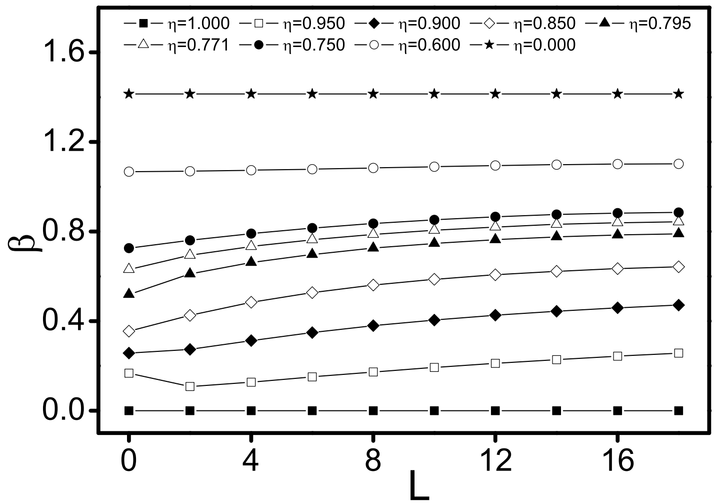

Figure 6: The phase diagram in terms of the deformation parameter β and the angular momentum L of the transitional nuclei in the between of U(5) and SU(3) symmetries.

U(5) symmetry is in good vibrational. For the transitional nucleus in the between of U(5) and SU(3) symmetries, it is like a soft liquid drop, whose shape may change to prolate ellipsoid and the deformation keeps increasing as the rotation gets rapid. Such an increase of deformation parameter with the increasing angular momentum is a corroboration of the well known variable moment of inertia[23]. Looking over Fig.6 more carefully, one may infer that, as the control parameter η takes a value about 0.8, the deformation parameter β changes with respect to the angular momentum most drastically. Then it may be around the critical point between the U(5) symmetry and the SU(3) symmetry. Referring Eq.(1), one knows that, the ratio between the interaction strength with U(5) symmetry (without two-body interactions being taken into account) and that with SU(3) symmetry $\frac{\eta}{(1-\eta)/N} \approx \frac{0.8}{0.2/10} = 40$. Recalling the results given in Refs.[14] and [24], such a ratio is just that of the critical symmetry X(5). Combining this point with Fig.5, one can infer that the ground state of the nucleus around the critical point may involve prolate and oblate shape coexistence, and its shape is so soft that it changes easily with the

increase of angular momentum. It is also remarkable that, for the states differing from those with U(5) symmetry very slightly (e.g., $\eta = 0.95$), the one with angular momentum $L = 2$ involves a smaller deformation parameter than its neighbors (e.g., with $L = 0, 4$). Since a smaller deformation parameter means a smaller rotational moment of inertia and contributes a relatively larger anharmonic effect (in rotation), such a phenomenon may be a manifestation of the 2_1^+ anomaly of near-vibrational states[25]. Up to now, a lot of works have been developed to remove the anomaly and to explore the mechanism(see for example Ref.[26] and references therein), the underlying physics remains to be studied further.

In conclusion, we have studied the shape phase structure and its evolution of the axially symmetric nuclear states in the between of the U(5) and the SU(3) symmetries in IBM-1. Especially the certain angular momentum effect or the rotation driven effect is studied by taking the angular momentum projection on the intrinsic coherent state. A phase diagram in terms of the deformation parameter and angular momentum is given. Meanwhile the case of $\beta < 0$, which corresponds to a oblate deformed shape, is included. The results show that there still can be a shape coexistence as the certain angular momentum effect is taken into account. However the coexistence is of oblate and prolate deformed shapes, but not of spherical and prolate shapes. Meanwhile, as the certain angular momentum effect is considered, the nucleus with SU(3) symmetry always appears as a rigid rotor with deformation parameter taking the classical limit value $\sqrt{2}$, and the one with U(5) symmetry as a pure vibrator. The nucleus in the transitional region between the U(5) symmetry and the SU(3) symmetry appears as a soft liquid drop, which involves both rotational and vibrational degrees of freedom. Then the ground state of some transitional nuclei may involve coexistence of prolate and oblate shapes. With the increasing of the angular momentum, the stable shape can only be prolate ellipsoid, whose deformation can get larger and larger. It also suggests that the states with the critical symmetry X(5) may be the one with prolate and oblate shape coexistence. Finally, it is remarkable that, the Hamiltonian we used here is very simple, the real case may be much more complicated. The related investigation is now under progress.

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References

[1] Y. Alhassid, S. Levit, and J. Zingman, Phys. Rev. Lett. **57** (1986), 5639; Y. Ahlassid, J. Zingman, and S. Levit, Nucl. Phys. **A 469** (1987), 205.

[2] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).

[3] F. Iachello, N. V. Zamfir, and R. F. Casten, Phys. Rev. Lett. **81** (1998),1191.

[4] J. Jolie, P. Cejnar, and J. Dobeš, Phys. Rev. **C 60** (1999), 061303(R).

[5] R. F. Casten, D. Kusnezov, and N. V. Zamfir, Phys. Rev. Lett. **82** (1999), 5000.

[6] F. Iachello, Phys. Rev. Lett. **87** (2001), 052502.

[7] N. Shimizu, T. Otsuka, T. Mizusaki, M. Honma, Phys. Rev. Lett. **86** (2001), 1171.

[8] P. Cejnar, Phys. Rev. **C 65** (2002), 044312.

[9] P. Cejnar P, Phys. Rev. Lett. **90** (2003), 112501.

[10] P. H. Regan, *et al.*, Phys. Rev. Lett. **90** (2003), 152502.

[11] F. Iachello, Phys. Rev. Lett. **91** (2003), 132502.

[12] A. Leviatan and J. N. Ginocchio, Phys. Rev. Lett. **90** (2003), 212501.

[13] P. Cejnar, S. Heinze, and J. Jolie, Phys. Rev. **C 68** (2003), 034326.

[14] F. Pan, J. P. Draayer, and Y. A. Luo, Phys. Lett. **B 576** (2003), 297.

[15] F. Iachello, and N. V. Zamfir, Phys. Rev. Lett. **92** (2004), 212501.

[16] P. Cejnar and J. Jolie, Phys. Rev. **C 69** (2004), 011301(R).

[17] N. Shimizu, T. Otsuka, T. Mizusaki, M. Honma, Phys. Rev. **C 70** (2004), 054313.

[18] P. Cejnar and J. Jolie, Phys. Rev. **E 58** (1998), 387.

[19] H. A. Lamme and E. Boeker, Nucl. Phys. **A 111** (1968), 492.

[20] J. Dobeš, Phys. Lett. **B 158** (1985), 96; Phys. Rev. **C 42** (1990), 2023.

[21] S. Kuyucak and I. Morrison, Phys. Rev. **C 36** (1987), 774.

[22] K. Hara, and Y. Sun, Int. J. Mod. Phys. **E 4** (1995), 637.

[23] M.A.J. Mariscotti, G. Scharff-Goldhaber, and B. Buck, Phys. Rev. **178** (1969), 1864.

[24] V. Werner, P. von Brentano, R.F. Casten, and J. Jolie, Phys. Lett. **B 527** (2002), 55.

[25] J. Kern, P. E. Garrett, J. Jolie, and H. Lemann, Nucl. Phys. **A 593** (1995), 21; J. Kern, and J. Jolie, Phys. Lett. **B 364** (1995), 207; J. Kern, and J. Jolie, Nucl. Phys. **A 624** (1997), 415; P. Cejnar, J. Jolie, and J. Kern, Phys. Rev. **C 63** (2001), 047304.

[26] M. A. Caprio, and R. F. Casten, and J. Jolie, Phys. Rev. **C 65** (2002), 034304.